HORIZONS

Conference in Honour of Petr Vopěnka

Book of Abstracts

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Praha 1

Joan Bagaria: *Vopěnka's Principle: from a Practical Joke to a True Axiom of Set Theory*

As related by Adámek and Rosický in their book on Locally Presentable and Accessible Categories, the story of Vopěnka's Principle is that of a practical joke that misfired. In the 1960's Vopěnka constructed (with Hedrlín and Pultr) a rigid graph on every set, and he thought that, with some more effort, one could also construct a rigid proper class of graphs. He then decided to tease set-theorists by introducing a new principle, known today as Vopěnka's Principle, and proved some consequences about large cardinals. He hoped that some set-theorists would continue his work (which they did) until somebody showed that the principle was nonsense. Today, more than 60 years later Vopěnka's Principle not only makes sense, but it is a principle of wide interest in set theory, with important applications. In this talk I will present some recent results showing why and how Vopěnka's Principle has acquired its prominent role in the theory of large cardinals.

Walter Dean: *On Skolem, Vopěnka, and the Ultrapower*

This talk will address how Vopěnka's reception of work of Skolem and Rieger informed his original application of the ultrapower construction to models of Gödel-Bernays set theory. I will begin by reconstructing the route by which Skolem (1929/33/34) was led to a form of the reduced product in his effort to construct a nonstandard model of True Arithmetic over a field of polynomials. I will then consider how Rieger (1959/61) adapted this technique as part of his attempt to define an arithmetical model of GB without the axiom of infinity and how he understood this to be related to the Continuum Hypothesis. I will finally address how Vopěnka (1964) built on these steps in his original proof of the independence of CH over GB and how this may have influenced the subsequent formulation of the Theory of Semisets (1972).

Randall Holmes: *Analogies between Jensen's Theory NFU with the*

Negation of Infinity and the Alternative Set Theory of Vopěnka

The subject of the talk will be formal analogies (and differences) between the alternative set theory of Vopenka on one hand and the variation of Quine's New Foundations due to Jensen augmented with the negation of the Axiom of Infinity, on the other. Both of these are theories which in some sense say that their worlds are finite, but which in fact have somewhat larger worlds. Both are theories of relatively low consistency strength with considerable expressive power for purposes of ordinary mathematics. In both cases, the applicability to normal mathematics involves some sort of thinking along the lines of nonstandard analysis, though neither theory really is exactly a system of nonstandard analysis.

Mikhail Katz: *Formalism 25*

Abraham Robinson's philosophical stance has been the subject of several recent studies.

Erhardt (2025) following Gaifman claims that Robinson was a finitist, and that there is a tension between his philosophical position and his actual mathematical output. We present evidence in Robinson's writing that he is more accurately described as adhering to the philosophical approach of Formalism. Furthermore, we show that Robinson explicitly argued against certain finitist positions in his philosophical writings. There is no tension between Robinson's mathematical work and his philosophy because mathematics and metamathematics are distinct fields: Robinson advocates finitism for metamathematics but no such restriction for mathematics. We show that Erhardt's analysis is marred by historical errors, by routine conflation of the generic and the technical meaning of several key terms, and by a philosophical parti pris. Robinson's Formalism remains a viable alternative to mathematical Platonism. (Joint work with Karl Kuhlemann and Semen Kutateladze.)

Karel Hrbacek: *Theory of Infinitesimals*

Starting with ideas that can be found in the writings of Leibniz and other early infinitesimalists, I will present a coherent, open-ended tower of axiomatic theories capable of formalizing almost all of contemporary nonstandard analysis. (Joint work with Mikhail G. Katz.)

Mauro di Nasso: *A Theory of "Numerosity" for Infinite Sets where the Whole is Larger than the Part*

The theory of numerosity was introduced to formalize the idea that counting the size of infinite sets is possible while maintaining the ancient principle that "the whole is greater than the part." In fact, contrary to Cantorian cardinality, the numerosity of a proper subset is always strictly smaller. A fundamental requirement is that numerosities form a discretely ordered ring and that sums and products correspond respectively to disjoint unions and Cartesian products, thus generalizing natural numbers as the numerosities of finite sets. The construction of models for numerosity has revealed several interesting foundational issues, as well as strong connections with nonstandard analysis. We will briefly discuss these issues, and also make some observations on the relationships and differences between numerosity and the notion of measure considered in mathematical analysis.

Gabriele Baratelli: *The "Symbolical" in Mathematics in Husserl's Early Work*

The talk has two main objectives. First, it aims to explain how the difficulties Husserl encountered in his early engagement with the philosophy of mathematics were decisive in the development of a new philosophical method, later called "phenomenology." Second, it will argue that the way he approached and addressed these conceptual challenges is relevant to clarifying some foundational concepts in mathematics, such as set and infinity. Together, these points highlight the paramount significance of the symbolic dimension of knowledge for understanding the mathematical sciences.

Mirja Hartimo: *Vopěnka on Evidenz*

This talk will first present the role of evidence [Evidenz] in Husserl's view of mathematics. According to it, mathematicians aim at constructing non-contradictory and at times also true statements or theories. When this activity is reflected upon transcendentally, it is conceived as a search for fulfilment in different kinds of evidence, in particular in distinctness [Deutlichkeit] and/or clarity [Klarheit]. Husserl's view of the kinds of evidence will then be used to examine Petr Vopěnka's view as presented in *New Infinitary Mathematics* (Karolinum Press, 2022). The aim is to find out the role of evidence in Vopěnka's view and how it compares to Husserl's view.

Štěpán Holub: *Vopěnka's Attempt to Reconstruct the Natural World*

While Vopěnka verbally professes Husserl's phenomenology, this talk will point out that his actual thinking deviates from transcendental phenomenology in crucial respects. The fundamental reason, I will argue, is that Vopěnka's guiding principle ultimately is not Husserl's philosophical "return to things themselves," but rather Bolzano's set-theoretical "explain any infinity as infinite multiplicity," which Vopěnka also endorses. The talk will outline how the tension between the two principles is reflected in Vopěnka's project of new infinitary mathematics. The key question is to what extent new infinitary mathematics, and especially the idea of the proper semiset, can shed light on the phenomenological problems of horizon and vagueness.

Gert Schubring: *Weierstraß's Unknown Elaboration of Complex Numbers as Foundation for a Rigorous Analysis*

In mathematics historiography, the 19th century is emphasised as the 'century of rigour', and in particlar in analysis. It used to be ascribed already to Cauchy to have achieved this rigour. The challenge for achieving rigour was to establish the concept of irrational numbers – a number field so far assumed only implicitly. Cauchy's 1821 textbook *Cours d'Analyse Algébrique* had, however, continued this practice. Its number concept did not provide the basis for modern analysis. Due to his substantialist understanding of numbers – only positive numbers were admitted as numbers, other domains being just 'quantities' or even only 'expressions' -, his conception did not provide that rigorous basis: the exclusion of zero led to complications in conceiving of limits, and understanding imaginary numbers only as expressions hampered to extend analysis beyond that of real functions.

Rigorous notions of irrational numbers were developed in the second half of the 19th century, by one French mathematician and three German mathematicians: Charles Méray, Georg Cantor, Richard Dedekind and Carl Weierstraß. Nowadays, it is mainly Dedekind's approach which is dominating in the understanding of real numbers, while

Weierstraß' conception is largely forgotten. Weierstraß had developed his conception not for establishing a rigorous notion of real numbers as is reported by all those who are aware of his achievements, but for a new notion of complex numbers – not just the "normal" notion of complex numbers as introduced by Gauß in 1831, but a more general one. This more general one, introduced first by French mathematicians in the 18th century and largely unknown today, served for Weierstraß as the number foundation for his favourite mathematical research, for elliptic functions.

The paper will focus on this conception of complex numbers.

Ladislav Kvasz: *Vopěnka on Visual Thinking in the History of Geometry: from Plato to Lobachevski*

Petr Vopěnka published under the title *Discourses with Geometry* (*Rozpravy s geometrií*) a four-volume treatise on the history of geometry. In it he, among other things, attempted to extend and develop into a fully-fledged description of the history of geometry the short account published by Edmund Husserl in the Revue Internationale de Philosophie in 1939 under the title “Die Frage nach dem Ursprung der Geometrie als intentional-historisches Problem”. Just like Husserl’s account, also its Vopěnka’s development starts and we may say, in a successful way by evoking the Platonic view of geometry. This view is not without problems, and Vopěnka does not hide its problems, but rather attempts their clear articulation. After developing the Platonic view of geometry Vopěnka turns to the discussion of its criticism by Aristotle, which in Vopěnka’s view led to the rise of the Euclidean position. As Vopěnka is the editor of the recent edition of the Czech translation of Euclid’s Elements, his knowledge of Euclid’s work is deep and detailed. So his comments on the differences between our modern interpretation of Geometry and that of Euclid are new and very interesting.

A further fascinating part of Vopěnka’s Discourses deals with the discussions of the developments in mathematics that occurred during the Middle Ages. Here, thanks to his deep insights into current theories of infinity, due to his active scientific works in axiomatic set theory, Vopěnka was able to give a radically new account of the medieval theories of infinity. Thus instead of viewing medieval mathematics as a period of stagnation or decline he emphasized the radical change that occurred with respect to the view of infinity. While the ancient Greek mathematicians viewed infinity as not suitable to mathematical investigation, in the Middle Ages this attitude was overcome. The third part of the Discourses is dedicated to the birth of analytic geometry. Vopěnka formulates an interesting thesis, according to which the straight lines were during the Renaissance extended to the infinity. And he characterizes early modern science, by using the analysis from Husserl's *Krisis*, as placing the world into the infinite geometrical space. In these two fundamental developments we see the impact of the medieval change of the attitude towards the infinite. The fourth and last part of the Discourses is focused on the birth of the non-Euclidean geometry. It is interesting that in this part Vopěnka changes philosophical preferences and instead of the Husserlian framework he interprets the birth of the non-Euclidean geometry in a Heideggerian way as a change of sensitivity (Befindligkeit).

Panel Discussion: *The Past and a Possible Future of Alternative Set Theory*

Roman Kossak (chair), Miroslav Holeček, Pavel Pudlák, Kateřina Trlifajová, Pavol Zlatoš